

### **HCT1.1: LINEAR ALGEBRA (4 Credits - 4 hours of Theory teaching per week)**

**Unit 1:** Fields, vector spaces, subspaces; linear dependence and independence; basis and dimension of a vector space, finite dimensional vector spaces completion theorem. Examples of vector spaces over real and complex fields. Linear equations. Vector spaces with an inner product, Gram-Schmidt orthogonalization process. Orthonormal basis and orthogonal projection of a vector.

**Unit 2:** Linear transformations, algebra of matrices, row and column spaces of a matrix. Elementary matrices, determinants, rank and inverse of a matrix. null space and nullity; partitioned matrices; Kronecker product. Hermite canonical form, generalized inverse, Moore-Penrose Inverse, Idempotent matrices. Solutions of matrix equations.

**Unit 3:** Triangular reduction of a positive definite matrix. Characteristic roots and vectors, Cayley-Hamilton theorem, minimal polynomial, similar matrices. Algebraic and geometric multiplicity of characteristic roots, spectral decomposition of a real symmetric matrix, reduction of a pair of real symmetric matrices, Hermitian matrices.

**Unit 4:** Real quadratic forms, reduction and classification of quadratic forms, index and signature. Singular values and singular decomposition, Jordan decomposition, extrema of quadratic forms. Vector and matrix differentiation.

#### **Books for Reference:**

- Bellman, R. (1970): Introduction to Matrix Analysis, Second Edition, McGraw Hill.
- Biswas, S. (1984): Topics in Algebra of Matrices, Academic Publications.
- Graybill, F. A. (1983): Matrices with Applications in Statistics, Second Edition, Wadsworth..
- Hadley, G. (1987): Linear algebra, Narosa.
- Halmos, P. R. (1958): Finite Dimensional Vector Spaces, Second Edition, D. Van Nostrand Company.
- Hoffman, K. and Kunze, R. (1971): Linear Algebra, Second Edition, Prentice Hall.
- Rao, A. R. and Bhimasankaram, P.(1992): Linear Algebra, Tata McGraw Hill.
- Rao, C. R (1973): Linear Statistical Inference and its Applications, Second Edition, Wiley.
- Rao, C. R. and Mitra, S. K (1971): Generalized Inverse of Matrices and its Applications, Wiley.
- Searle, S. R (1982): Matrix Algebra Useful for Statistics, Wiley.

**HCT 1.2: REAL ANALYSIS (4 Credits – 4 hours of Theory teaching per week)**

**Unit 1:** Elements of set theory, Sets in Euclidean space of  $k$ -dimensional  $\mathbb{R}^k$  rectangles, neighbourhood, interior point and limit point, open and closed sets, Bolzano-Weierstrass theorem in  $\mathbb{R}^2$ , Real valued functions continuity and uniform continuity.

**Unit 2:** Sequences and Series of constants- Limit superior, limit inferior and limit - properties. Cesaro sequences. Series of positive terms - Tests for convergence, divergence. Integral and Order tests and Kummer's test (statement only of all the tests)- Ratio and Raabe's tests as special cases of Kummer's test. Series of arbitrary terms - absolute and conditional convergence.

**Unit 3:** Sequences of functions-Uniform convergence and point wise convergence, Series of functions-uniform convergence-Weierstrass' M test. Power series and radius of convergence. Riemann-Stieltjes integration-continuous integrand and monotonic /differentiable integrator.

**Unit 4:** Functions of two variables-partial and directional derivatives. Maxima and minima of functions, maxima-minima under constraints (Lagrange's multipliers).

**Unit 5:** Parametric functions. Uniform convergence of improper integrals, Differentiation under integrals. Double integrals and repeated integrals. Change of variables under double integration-statement of the theorems without proof and solution of problems.

**Books for reference:**

Apostol, T.M. (1985): Mathematical Analysis, Narosa India Ltd.

Courant, R. and John, F. (1965): Introduction to Calculus and Analysis, Wiley.

Goldberg, R.R.(1970): Methods of Real Analysis, Oxford Publishing Co.

Khuri, A.T. (1993): Advanced Calculus with Applications in Statistics, John Wiley.

Rudin, W. (1976): Principles of Mathematical Analysis, Mc Graw Hill.

Shantinarayan (1950) : A course on Mathematical analysis, Sultan Chand and Co.